Section 5.3

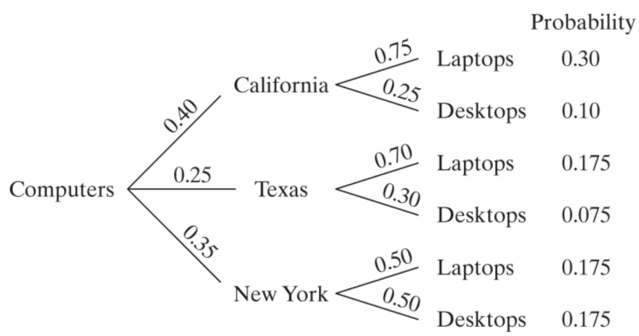
Check Your Understanding, page 321:

1. $P(L) = \dfrac{2268 + 800 + 588}{10{,}000} = \dfrac{3656}{10{,}000} = 0.3656$. There is a 0.3656 probability of selecting a course grade that is lower than a B.

2. $P(E \mid L) = \dfrac{800}{3656} = 0.219$. $P(L \mid E) = \dfrac{800}{368 + 432 + 800} = \dfrac{800}{1600} = 0.50$. To answer the question about whether engineering students earn lower grades than students in the liberal arts, you would want to look at $P(L \mid E)$. This gives the probability of getting a lower grade given that the student is studying engineering or physical science. Because this probability (0.50) is greater than $P(L) = 0.3656$, we can conclude that grades are lower in engineering and physical sciences.

Check Your Understanding, page 326:

1.



2.
$P(\text{laptop}) = P(\text{CA} \cap \text{laptop}) + P(\text{TX} \cap \text{laptop}) + P(\text{NY} \cap \text{laptop}) = 0.30 + 0.175 + 0.175 = 0.65.$
There is a 0.65 probability that the computer is a laptop.

3. $P(\text{made in CA} \mid \text{laptop}) = \dfrac{P(\text{made in CA} \cap \text{laptop})}{P(\text{laptop})} = \dfrac{0.30}{0.30 + 0.175 + 0.175} = \dfrac{0.30}{0.65} = 0.462.$
Given that a laptop was selected, there is a 0.462 probability that it was made in CA.

Check Your Understanding, page 328:

1. *A* and *B* are independent. Because we are putting the first card back and then re-shuffling the cards before drawing the second card, knowing what the first card was will not help us predict what the second card will be.
2. *A* and *B* are not independent. Once we know the suit of the first card, then the probability of getting a heart on the second card will change depending on what the first card was.
3. The two events, "female" and "right-handed" are independent. Once we know that the chosen person is female, this does not help us predict if she is right-handed or not. Overall, $\dfrac{24}{28} = \dfrac{6}{7}$ of the students are right-handed. And, among the women, $\dfrac{18}{21} = \dfrac{6}{7}$ are right-handed. So *P*(right-handed) = *P*(right-handed | female).

Check Your Understanding, page 331:

1. $P(\text{returned safely}) = 1 - P(\text{was lost}) = 1 - 0.05 = 0.95$. So, if there are 20 missions and the outcomes of missions are independent,

$P(\text{safe return on all 20 missions}) = P(\text{1st safe})P(\text{2nd safe})\ldots P(\text{20th safe}) = 0.95^{20} = 0.3585$.
There is a 0.3585 probability that the bomber returned safely from 20 missions.

2. No, we cannot conclude that 2.4% of adults 55 or older are college students. Being a college student and being 55 or older are not independent events. $P(\text{college} \mid \text{55 or older})$ is much smaller than $P(\text{college})$.

Exercises, page 333:

5.63 (a) $P(\text{almost certain} \mid \text{M}) = \frac{597}{2459} = 0.2428$. Given that the person selected is male, there is a 0.2428 probability that he answered "almost certain."

(b) $P(\text{F} \mid \text{Some chance}) = \frac{426}{712} = 0.5983$. Given that the person selected said "some chance but probably not," there is a 0.5983 probability that the person is female.

5.64 (a) $P(\text{survived} \mid \text{first class}) = \frac{197}{197+122} = \frac{197}{319} = 0.6176$. Given that the person selected was in first class, there is a 0.6176 probability that he or she survived.

(b) $P(\text{third class} \mid \text{survived}) = \frac{151}{197+94+151} = \frac{151}{442} = 0.3416$. Given that the person selected survived, there is a 0.3416 probability that he or she was a third-class passenger.

5.65 (a) $P(D \mid F) = \frac{13}{13+4} = \frac{13}{17} = 0.7647$. Given that a Senator is female, there is a 0.7647 probability that she is a Democrat.

(b) $P(F \mid D) = \frac{13}{47+13} = \frac{13}{60} = 0.2167$. Given that a Senator is a Democrat, there is a 0.2167 probability that she is a female.

5.66 (a) $P(B \mid M) = \frac{190}{320} = 0.5938$. Given that a student is male, there is a 0.5938 probability that he eats breakfast regularly.

(b) $P(M \mid B) = \frac{190}{300} = 0.6333$. Given that a student is a regular breakfast eater, there is a 0.6333 probability that he is a male.

5.67 (a) $P(\text{is studying other than English}) = 1 - P(\text{none}) = 1 - 0.59 = 0.41$. There is a 0.41 probability that the student is studying a language other than English.

(b) $P(\text{Spanish} \mid \text{other than English}) = \frac{0.26}{0.41} = 0.6341$. Given that the student is studying some language other than English, there is a 0.6341 probability that he or she is studying Spanish.

5.68 (a) $P(\$50{,}000 \text{ or more}) = 0.215 + 0.100 + 0.006 = 0.321$. There is a 0.321 probability that a randomly chosen return shows an adjusted gross income of $50,000 or more.

(b) $P(\text{at least \$100,000} \mid \text{at least \$50,000}) = \dfrac{0.106}{0.321} = 0.3302$. Given that the return shows an income of at least \$50,000, there is a 0.3302 probability that the income is at least \$100,000.

5.69 $P(B) < P(B \mid T) < P(T) < P(T \mid B)$. There are very few professional basketball players, so $P(B)$ should be the smallest probability. If you are a professional basketball player, it is quite likely that you are tall, so $P(T \mid B)$ should be the largest probability. Finally, it's much more likely to be over 6 feet tall than it is to be a professional basketball player if you're over 6 feet tall.

5.70 $P(T) < P(T \mid A) < P(A) < P(A \mid T)$. Nearly everyone whose career is teaching has a college degree so $P(\mathrm{A} \mid \mathrm{T})$ will have the largest probability (close to 1). A higher percentage of the general population will have college degrees than will be teachers, so $P(T) < P(A)$. Finally, the proportion of teachers among the college educated is smaller than the proportion of college educated among all people, so $P(T \mid A) < P(A)$.

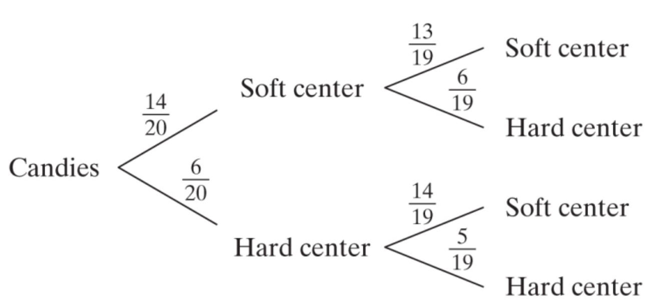
5.71 $P(\text{YT} \mid \text{FB}) = \dfrac{P(\text{YT} \cap \text{FB})}{P(\text{FB})} = \dfrac{0.66}{0.85} = 0.7765$. Given that the student uses Facebook regularly, there is a 0.7765 probability that the student also uses YouTube regularly.

5.72 $P(G \mid PC) = \dfrac{P(G \cap PC)}{P(PC)} = \dfrac{0.23}{0.60} = 0.383$. Given that a student mainly uses a PC, there is a 0.383 probability that the person is a graduate student.

5.73 P(download music) = 0.29, P(don't care | download music) = 0.67.
$P(\text{download music} \cap \text{don't care}) = P(\text{download music}) \cdot P(\text{don't care} \mid \text{download music})$
$= (0.29)(0.67) = 0.1943 = 19.43\%$. About 19% of internet users download music and don't care if it is copyrighted.

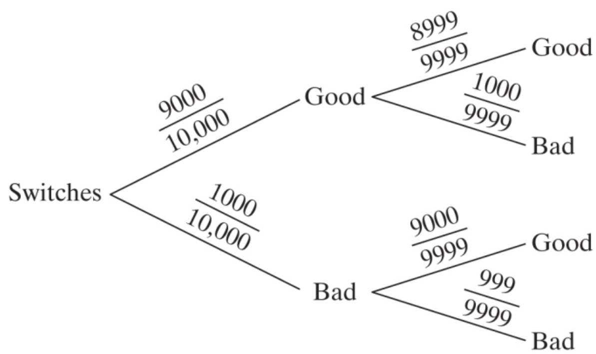
5.74 P(belong to health club) = 0.10, P(go twice/week | belong) = 0.40.
$P(\text{belong to health club} \cap \text{go twice a week}) = P(\text{belong})P(\text{go twice/week} \mid \text{belong})$
$= (0.40)(0.10) = 0.04 = 4\%$. About 4% of adults belong to a health club and go at least twice a week.

5.75 (a) A tree diagram is below.



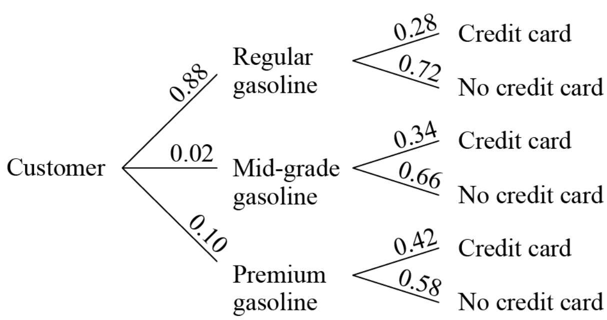
(b) $P(\text{one soft} \cap \text{one hard}) = \left(\frac{14}{20}\right)\left(\frac{6}{19}\right) + \left(\frac{6}{20}\right)\left(\frac{14}{19}\right) = \frac{84}{380} + \frac{84}{380} = \frac{168}{380} = 0.4421$. There is a 0.4421 probability that one of the chocolates has a soft center and the other one doesn't.

5.76 (a) A tree diagram is below.



(b) $P(\text{defective} \cap \text{defective}) = \left(\frac{1000}{10{,}000}\right)\left(\frac{999}{9999}\right) = \frac{999{,}000}{99{,}990{,}000} = 0.00999$. There is a 0.00999 probability that both switches are defective.

5.77 (a) A tree diagram is below.

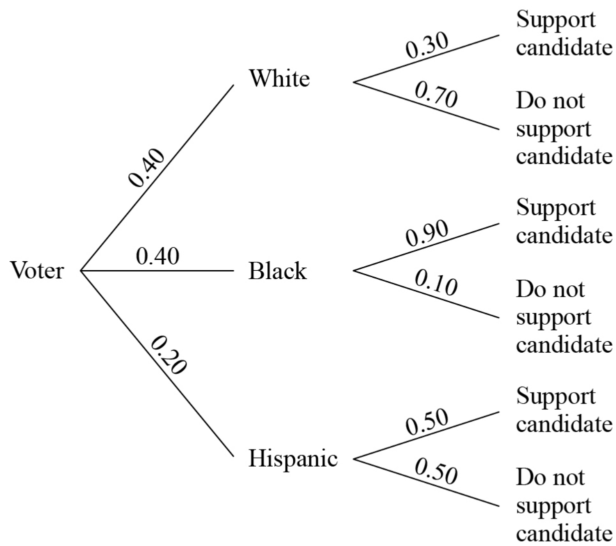


(b) P(credit card) = (0.88)(0.28) + (0.02)(0.34) + (0.10)(0.42) = 0.2464 + 0.0068 + 0.0420 = 0.2952. There is a 0.2952 probability that the customer paid with a credit card.

(c) $P(\text{premium gasoline} \mid \text{credit card}) = \frac{P(\text{premium gasoline} \cap \text{credit card})}{P(\text{credit card})} = \frac{0.0420}{0.2952} = 0.142$.

Given that the customer paid with a credit card, there is a 0.142 probability that the customer bought premium gas.

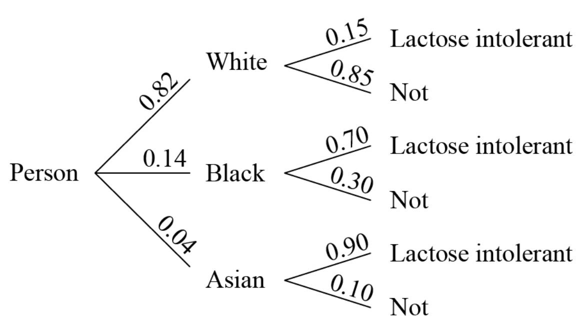
5.78 (a) A tree diagram is below.



(b) *P*(support) = (0.40)(0.30) + (0.40)(0.90) + (0.20)(0.50) = 0.12 + 0.36 + 0.10 = 0.58. There is a 0.58 probability that this voter supports the mayoral candidate.

(c) $P(\text{black} \mid \text{support}) = \dfrac{P(\text{black} \cap \text{support})}{P(\text{support})} = \dfrac{0.36}{0.58} = 0.6207$. Given that the voter supports the mayoral candidate, there is a 0.6207 probability that the voter is black.

5.79 (a) Here is a tree diagram.



P(lactose intolerant) = (0.82)(0.15) + (0.14)(0.70) + (0.04)(0.90) = 0.123 + 0.098 + 0.036 = 0.257. There is a 0.257 probability that a randomly selected U.S. person is lactose intolerant.

(b) $P(\text{Asian} \mid \text{lactose intolerant}) = \dfrac{P(\text{Asian} \cap \text{lactose intolerant})}{P(\text{lactose intolerate})} = \dfrac{0.036}{0.257} = 0.1401$. Given that a randomly selected person is lactose intolerant, there is 0.1401 probability that the person is Asian.

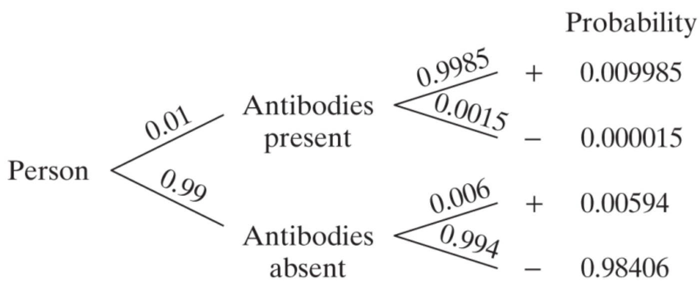
5.80 (a)

$$P(\text{contributed}) = (0.5)(0.4)(0.8) + (0.3)(0.3)(0.6) + (0.2)(0.1)(0.5) = 0.16 + 0.054 + 0.01 = 0.224.$$

There is a 0.224 probability that a potential donor contributed to the charity.

(b) $P(\text{recent donor} \mid \text{contribute}) = \dfrac{P(\text{recent donor} \cap \text{contribute})}{P(\text{contribute})} = \dfrac{0.16}{0.224} = 0.7143$. Given that the person contributes to the charity, there is a 0.7143 probability that the person was a recent donor.

5.81 Here is a tree diagram to help organize the given information.



$$P(\text{antibody} \mid \text{positive}) = \frac{P(\text{antibody} \cap \text{positive})}{P(\text{positive})} = \frac{(0.01)(0.9985)}{(0.01)(0.9985)+(0.99)(0.006)}$$

$$= \frac{0.009985}{0.009985+0.00594} = \frac{0.009985}{0.015925} = 0.6270.$$

Given that the EIA test is positive, there is a 0.6270 probability that the person has the antibody.

5.82 (a) Here is a two-way table to help organize the information.

	Have condition	**Do not have condition**	**Total**
Test is positive	240	50	**290**
Test is negative	10	700	**710**
Total	**250**	**750**	**1000**

A false-positive means that the technician identified someone as having the condition when, in fact, they do not have the condition. This happened in 50 out of 750 healthy people so the false-positive rate was $\dfrac{50}{750} = 0.0667$. A false-negative means that the technician identified someone as not having the condition when, in fact, they have the condition. This happened in 10 out of 250 patients so the false-negative rate was $\dfrac{10}{250} = 0.04$.

(b) P(have condition | positive test result) = 240/290 = 0.8276. Given that a patient got a positive test result, there is a 0.8276 probability that the patient has the medical condition.

5.83 (a) $P(\text{a good chance} \mid \text{female}) = \dfrac{663}{2367} = 0.2801$. Given that a person is female, there is a 0.2801 probability that the person will respond "a good chance."

(b) $P(\text{a good chance}) = \dfrac{1421}{4826} = 0.2944$. There is a 0.2944 probability that the person will respond "a good chance."

(c) The events "a good chance" and "female" are not independent because the two probabilities in parts (a) and (b) are not the same. Knowing that a person is a female changes the probability that the person will respond "a good chance."

5.84 (a) $P(\text{survived} \mid \text{second class}) = \dfrac{94}{94+167} = \dfrac{94}{261} = 0.3602$. Given that a person was in second class, there is a 0.3602 probability that the person survived.
(b) $P(\text{survived}) = \dfrac{197+94+151}{197+122+94+167+151+476} = \dfrac{442}{1207} = 0.3662$. There is a 0.3662 probability that the person survived.
(c) The events "survived" and "second class" are not independent because the two probabilities in parts (a) and (b) are not the same. Knowing that a person was in second class changes the probability that the person survived.

5.85 $P(D) = \dfrac{60}{100} = 0.60$. From exercise 5.65 we saw that $P(D \mid F) = 0.7647$. Because these two probabilities are not the same, events *D* and *F* are not independent. Knowing that a senator is female changes the probability that the senator is a democrat.

5.86 $P(B) = \dfrac{300}{595} = 0.504$. From exercise 5.66 we saw that $P(B \mid M) = 0.5938$. Because these two probabilities are not the same, events *B* and *M* are not independent. Knowing that a student is male changes the probability that the student eats breakfast regularly.

5.87 There are 36 different possible outcomes of the two dice: (1,1), (1,2),…,(6,6). Let's assume that the second die is the green die. There are then six ways for the green die to show a 4: (1,4), (2,4), (3,4), (4,4), (5,4), (6,4). Of those, there is only one way to get a sum of 7, so $P(\text{sum of } 7 \mid \text{green is } 4) = \dfrac{1}{6} = 0.1667$. Overall, there are 6 ways to get a seven: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1). So $P(\text{sum of } 7) = \dfrac{6}{36} = 0.1667$. Because these two probabilities are the same, the events "sum of 7" and "green die shows a 4" are independent. Knowing that the green die shows a 4 does not change the probability that the sum is 7.

5.88 There are 36 different possible outcomes of the two dice: (1,1), (1,2),…,(6,6). Let's assume that the second die is the green die. There are then six ways for the green die to show a 4: (1,4), (2,4), (3,4), (4,4), (5,4), (6,4). Of those, there is only one way to get a sum of 8, so $P(\text{sum of } 8 \mid \text{green is } 4) = \dfrac{1}{6} = 0.1667$. Overall, there are 5 ways to get an eight: (2,6), (3,5), (4,4), (5,3), (6,2). So $P(\text{sum of } 8) = \dfrac{5}{36} = 0.1389$. Because these two probabilities are not the same, the events "sum of 7" and "green die shows a 4" are not independent. Knowing that the green die shows a 4 changes the probability that the sum is 8.

5.89 An individual light remains lit for 3 years with probability 1 − 0.02 = 0.98. The whole string remains lit with probability $(0.98)^{20} = 0.6676$.

5.90 The probability that a single author does not have one of the 10 most common names is 1 – 0.096 = 0.904. The probability that all 9 authors do not have one of the 10 most common last names is $P(\text{none common}) = (0.904)^9 = 0.4032$. Because this probability is fairly large, we should not be surprised if none of the names of these authors was among the 10 most common.

5.91 The probability that any one person is not a universal donor is $1 - 0.072 = 0.928$. So $P(\text{none are universal donors}) = (0.928)^{10} = 0.4737$. This means that $P(\text{at least one universal donor}) = 1 - 0.4737 = 0.5263$. There is a 0.5263 probability that at least one of the 10 Americans will be a universal donor.

5.92 The probability that any one Internet reference works in two years is 1 – 0.13 = 0.87. So, P(all work in two years) = $(0.87)^7$ = 0.3773. This means that P(at least one doesn't work) = 1 – 0.3773 = 0.6227. There is a 0.6227 probability that at least one of the seven Internet references won't work two years later.

5.93 We cannot simply multiply the probabilities together. If the first of the three consecutive shows starts late, it is much less likely that the next show will start on time. These events are not independent of each other.

5.94 We cannot simply multiply the probabilities together. It is likely that if one flight is late, that whatever is causing it to be late (weather, backed-up airplanes at the airport, etc) will also be affecting the other three flights. These four events are not independent of each other.

5.95 (a) There are 6 ways to get doubles out of 36 possibilities so $P(\text{doubles}) = \frac{6}{36} = \frac{1}{6} = 0.167$.
There is a 0.167 probability of getting doubles.
(b) Because the rolls are independent we can use the multiplication rule for independent events.
$P(\text{no doubles first} \cap \text{doubles second}) = P(\text{no doubles first})P(\text{doubles second})$

$$= \frac{30}{36}\left(\frac{6}{36}\right) = \frac{5}{6}\left(\frac{1}{6}\right) = \frac{5}{36} = 0.139.$$

There is a 0.139 probability of not getting doubles on the first toss and getting doubles on the second toss.
(c)

$$P(\text{first doubles on third roll}) = P(\text{no doubles})P(\text{no doubles})P(\text{doubles}) = \frac{5}{6}\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) = \frac{25}{216} = 0.116.$$

There is a 0.116 probability that the first doubles occurs on the third roll.

(d) For the first doubles on the fourth roll, the probability is $\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)$. For the first doubles on the fifth roll, the probability is $\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$. The general result is that the probability that the first doubles are rolled on the kth roll is $\left(\frac{5}{6}\right)^{k-1}\left(\frac{1}{6}\right)$.

5.96 (a) $P(1^{st}\text{ card } \spadesuit) = \frac{13}{52} = 0.25$, $P(2^{nd}\text{ card } \spadesuit \mid \spadesuit\text{ picked}) = \frac{12}{51} = 0.2353$.

(b) $P(3^{rd}\text{ card } \spadesuit \mid 2\spadesuit\text{s picked}) = \frac{11}{50} = 0.22$, $P(4^{th}\text{ card } \spadesuit \mid 3\spadesuit\text{s picked}) = \frac{10}{49} = 0.2041$, $P(5^{th}$ card $\spadesuit \mid 4\spadesuit\text{s picked}) = \frac{9}{48} = 0.1875$.

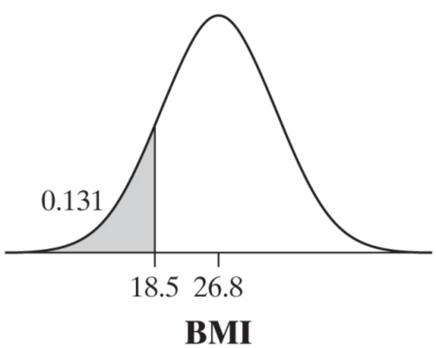
(c) The product of these conditional probabilities gives the probability of a flush in spades by the extended multiplication rule: We must draw a spade, and then another, and then a third, a fourth, and a fifth. The product of these probabilities is $\left(\frac{13}{52}\right)\left(\frac{12}{51}\right)\left(\frac{11}{50}\right)\left(\frac{10}{49}\right)\left(\frac{9}{48}\right) = 0.0004952$. There is about a 0.0005 probability of being dealt 5 spades in a row.
(d) Because there are four possible suits in which to have a flush, the probability of a flush is four times the probability found in (c), or about 4(0.0004952) = 0.001981. There is a 0.001981 probability of being dealt a flush.

5.97 c

5.98 e

5.99 e

5.100 **Step 1: State the distribution and values of interest.** BMI for American young women follows a Normal distribution with mean 26.8 and standard deviation 7.4. We want to find the percent of women with BMI less than 18.5 (see graph below). **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{18.5 - 26.8}{7.4} = -1.12$. From Table A, the proportion of z-scores below –1.12 is 0.1314. *Using technology:* The command normalcdf(lower: –1000, upper: 18.5, μ: 26.8, σ: 7.4) gives an area of 0.1310. **Step 3: Answer the question.** About 13% of American young women are classified as underweight.



5.101 Note that $P(\text{not underweight}) = 1 - P(\text{underweight}) = 1 - 0.131 = 0.869$.
$P(\text{at least one is underweight}) = 1 - P(\text{none are underweight}) = 1 - 0.869^2 = 0.2448$. There is a 0.2448 probability that at least one of the two women will be classified as underweight.

5.102 There appears to be an association in the population of adults. Generally, more education means more freedom. For example, 45% of those with bachelor's degrees felt free to organize their work, compared to 29.7% of those who finished high school and 24.4% of those who did not finish high school. This can also be seen in the bar graph below.